CS581 – Theory of Computation – HW9

Tuesday, June 4, 2013
due in class Tuesday, June 11, 2013

Answer each question below. You will turn this homework in using D2L. In addition, you may also turn in a paper copy in class. In this case the TA will mark up your homework with comments and return the comments to you.

You may format your answers using some document processing software, or you may write it up with pencil and paper and scan it. In either case submit a pdf document. Be sure your submission is clearly identified as Homework 8, and contains your name and your email on the first line. The first line should look like:

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1. Fill out the table described in the notes (and on page 263 of the text) for the polynomial time algorithm for context free language recognition, for the string $w = baba$ and the CFG G:
   - $S \rightarrow RT$
   - $R \rightarrow TR \mid a$
   - $T \rightarrow TR \mid b$
2. Show that NP is closed under union and concatenation.
3. A boolean formula is in conjunctive normal form (called a cnf-formula) if it comprises several clauses $\wedge$-ed together, and each clause is a sequence of literals joined by $\vee$. A literal is a variable or a negated variable. An example is

   $$(x_1 \vee x_2) \wedge (x_3 \vee \overline{x_4} \vee x_5 \vee \overline{x_6})$$

   A formula is in 3cnf-form if all the clauses have exactly 3 literals. Show that 3SAT $= \{x \mid \text{x is a satisfiable 3cnf-formula}\}$ is NP-complete by polynomial time reducability from SAT (I.e. show that a known NP-complete problem, SAT, can be reduced to 3SAT).

4. A *triangle* in an undirected graph is just a *3-clique*. We know that *n-clique* $\in$ NP. Show that $TRIANGLE \in$ P, where TRIANGLE $= \{< G >$ $|$ G contains a triangle$\}$.

5. Show that if $P = NP$, then every language $A \in P$, except $A = \emptyset$ and $A = \Sigma^*$ is NP-complete. Hint: recall the definition of NP-complete.

6. Let G represent an undirected graph. Also let

   - SPATH $= \{< G, a, b, k >$ $|$ G contains a simple path of length *at most* k from a to b$\}$
   - LPATH $= \{< G, a, b, k >$ $|$ G contains a simple path of length *at least* k from a to b$\}$

   Show the following:

   - Show that $SPATH \in$ P.
   - Show that $LPATH \in$ NP.